Int. J. Therm. Sci. (1999) 38, 525-529

Optimum design of vertical rectangular fin arrays

Andrea de Lieto Vollaro*, Stefano Grignaffini, Franco Gugliermetti

University of Rome 'La Sapienza', Department of 'Fisica Tecnica', Via Eudossiana, 18, 00184 Rome, Italy

(Received 22 July 1998, accepted 16 November 1998)

Abstract — The goal of this analysis has been the search for the optimal configuration for a finned plate (with rectangular and vertical fins) to be cooled in natural convection. Utilizing a simplified relation of the fins heat exchange some simple expressions for the determination of the optimum value of the fins spacing have been developed as a function of the parameters which feature in the configuration: dimensions, thermal conductivity, fins absorption coefficient and fluid thermo-physical properties. © Elsevier, Paris.

Heat sinks / rectangular fin arrays / optimum value of the fins spacing / natural convection

Résumé — Conception optimale d'une rangée d'ailettes rectangulaires verticales de radiateur. Ce travail concerne la recherche de la configuration optimale de radiateurs à ailettes rectangulaires et verticales, refroidis par convection naturelle. Pour l'essentiel, on détermine, en utilisant une relation simplifiée pour l'échange thermique, la valeur optimale du pas d'ailettes. La recherche de cette valeur optimale prend en compte les différents paramètres thermiques et géométriques du radiateur : dimensions, conductivité thermique, propriétés thermiques et physiques du fluide, ainsi que le rayonnement dans les cavités formées par les ailettes. © Elsevier, Paris.

ailettes rectangulaires et verticales / pas optimal entre ailettes / convection naturelle

Nomenclature

a_1, a_2	absorption coefficient of the fins surfaces	
a, c	constants in equations (5), (6), (10), (13), (15)	
b	fin length (see *figure 1*)	m
c_p	specific heat at constant pressure	$\text{kJ·kg}^{-1}\text{·K}^{-1}$
d	fin spacing (see *figure 1*)	m
$F_{31}, F_{22}, F_{12}, F_{32}, F_{12}$	two-dimensional view factors of a rectangular cavity	
g	acceleration due to gravity	m·s^{-2}
H	overall heat transfer coefficient	$\text{W·m}^{-2}\text{·K}^{-1}$
h	average heat transfer coefficient	$\text{W·m}^{-2}\text{·K}^{-1}$
L	fin height	m
N	number of fins	
Nu	Nusselt number $\left(= \frac{h\,d}{\lambda_a}\right)$	
Q	heat flux exchanged by the finned surface	W·m^{-2}
Ra	Rayleigh number $\left(= Gr_b\,Pr = \frac{g\,\beta\,\Delta T\,b^3\,\delta^2\,c_p}{\mu\,\lambda_a}\right)$	
Ra'	modified Rayleigh number $\left(= Gr_d\,Pr\frac{d}{b} = \frac{g\,\beta\,\Delta T\,d^3\,\delta^2\,c_p}{\mu\,\lambda_a}\,\frac{d}{b}\right)$	
r_1, r_2	reflection coefficient of the fins surfaces	
T	temperature	°C
ΔT	temperature difference	K
t	fin thickness (see *figure 1*)	m
z	width of base plate (see *figure 1*)	m
x	non-dimensional fins spacing	

Greek symbols

δ	fluid density	kg·m^{-3}
η	fin efficiency	
λ	thermal conductivity	$\text{W·m}^{-1}\text{·K}^{-1}$
μ	fluid viscosity	$\text{kg·m}^{-1}\text{·s}^{-1}$

Subscripts

b	referred to the base plate
∞	referred to the unperturbed fluid
f	referred to the fin material
a	referred to the fluid
opt	optimum value

* Correspondence and reprints.
delietov_a@pcfite.ing.uniroma1.it

1. INTRODUCTION

One of the most important aspects of electronic equipment management has always been recognised to be the dissipation of the heat produced in the electronic components, which is needed to avoid overheating of the apparatus.

During the years the electronic equipment cooling problem has become even more crucial, as a consequence of the continuous evolution reached by the electronics industry year by year, creating apparatus even more compact in their dimensions. Consequently the quantity of heat to be dispersed is higher per unit area. With the above in mind it is clear the importance to be given to the optimization of the cooling systems of the electronic equipment.

The heat transfer to the external ambient atmosphere by the electronic apparatus can be obtained mainly by using the mechanisms of the heat transfer by forced convection, natural convection and by radiative heat transfer. This paper deals with those issues related to the cooling obtained only by natural convection and radiative heat transfer. The goal of this analysis has been the search for the optimal configuration for a finned plate (with rectangular and vertical fins) to be cooled as shown in *figure 1.*

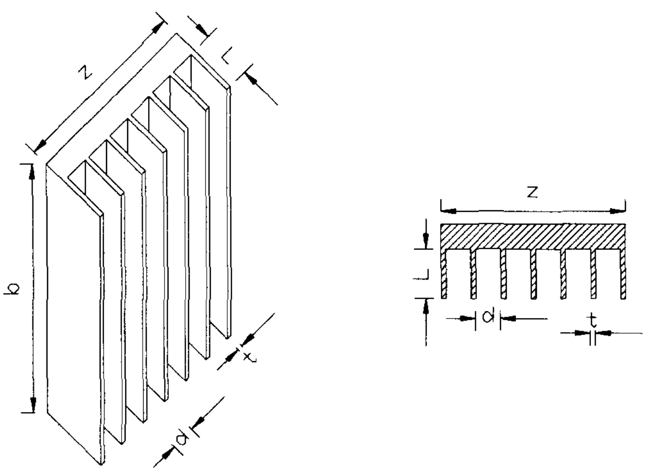

Figure 1. Axonometric projection and cross-sectional view of the finned plate with rectangular and vertical fins.

This problem has been already dealt with in [1, 2, 3, 4, 5], taking into account, at least in one case [3], the effect on the optimal configuration of the thermal conductivity of the material used to manufacture the fins.

Utilising a simplified relation for the fins efficiency some simple expressions for the determination of the optimal value of the fins spacing have been developed, in direct relation with the parameters featuring in the configuration: dimensions, thermal conductivity, fins absorption coefficient, thermo-physical properties of the fluids.

The formulas utilised have permitted us to verify the conditions at which it is possible to elaborate a simplified calculation in a quicker and simpler way.

2. CONVECTIVE AND RADIATIVE HEAT TRANSFER FROM INFINITE FIN ARRAYS

If t and d are much smaller than z, it can be assumed that the number of fins is:

$$N = \frac{z}{(t+d)} + 1 \qquad (1)$$

where +1 is negligible.

It will be assumed that the system is fully two-dimensional, that is, the height b is considered much larger than the other lengths and surface temperature variations along the vertical direction are negligible.

An overall heat transfer coefficient, H, is defined as the ratio of the heat flux exchanged by the finned surface, Q, and the product $z\,b\,(T_b - T_\infty)$ of the base surface, $z\,b$, and the difference, $T_b - T_\infty$, between base temperature and ambient temperature:

$$H = \frac{Q}{(T_b - T_\infty)\,z\,b} = \frac{z\,d\,b}{(t+d)}\,\frac{h}{z\,b} + \frac{2\,N\,L\,b\,h\,\eta}{z\,b} \qquad (2)$$

where η is the fin efficiency defined as:

$$\eta = \frac{1}{L}\,\frac{\int_0^L (T - T_\infty)\,dL}{(T_b - T_\infty)} \qquad (3)$$

To evaluate the heat transfer coefficient of the finned surface, the following expression of Nu is assumed [1, 2]:

$$Nu = \frac{h\,d}{\lambda_a} = \frac{1}{\sqrt{\dfrac{576}{(Ra')^2} + \dfrac{2{,}873}{\sqrt{Ra'}}}} \qquad (4)$$

with $Ra' = Gr_d\,Pr\,\dfrac{d}{b} = \dfrac{g\,\beta\,\Delta T\,d^3\,\delta^2\,c_p}{\mu\,\lambda_a}\,\dfrac{d}{b}$

From (4) the following expression for h in terms of the Rayleigh number $Ra = Gr_b\,Pr$ and of the non-dimensionalized spacing $x = \dfrac{d\,\sqrt[4]{Ra}}{b}$ is obtained:

$$\frac{h\,b}{\lambda_a} = \frac{x\,\sqrt[4]{Ra}}{\sqrt{\left(\dfrac{a}{x^4} + c\,x^2\right)}} \qquad (5)$$

with $a = 576$; $c = 2{,}873$; the overall heat transfer coefficient becomes:

$$\frac{H\,b}{\lambda_a} = \frac{2\,L'\,\sqrt[4]{Ra}}{\left(1 + \dfrac{t'}{x}\right)\sqrt{\dfrac{a}{x^4} + c\,x^2}}\left(\eta + \frac{x}{2\,L'}\right) \qquad (6)$$

where:

$$L' = \frac{L}{b}\sqrt[4]{Ra};\ t' = \frac{t}{b}\sqrt[4]{Ra} \qquad (7)$$

The radiative contribution to the total heat flux is calculated from the hypothesis that the channel between two adjacent fins can be treated as a cavity of infinite height bounded by three grey surfaces, and one black surface (the open side) at T_∞ (*figure 2*).

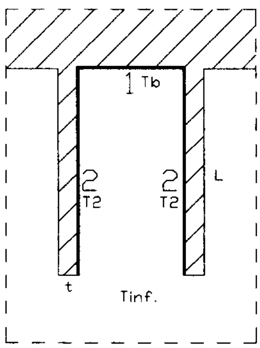

Figure 2. Detail of the cross-sectional view of a U-shaped unit.

The term that must be added to (6) is:

$$Hr\,\frac{b}{\lambda_a} = \frac{z\,d\,b}{(t+d)}\,\frac{1}{z\,b}\left\{\begin{array}{l}\dfrac{F_{31}}{B}\left[a_1\,\varepsilon_{rb}\,(1-r_2\,F_{22}) + 2\,F_{12}\,a_2\,\varepsilon_{r2}\,r_1\right]\\ +\dfrac{2\,F_{32}}{B}\left[a_2\,\varepsilon_{r2}\,\eta + F_{21}\,a_1\,\varepsilon_{rb}\right]\end{array}\right\} \qquad (8)$$

where F_{31},F_{22},F_{12},F_{32},F_{12}, are the two-dimensional view factors of a rectangular cavity, and:

$$\varepsilon_{rb} = 4\,\sigma_0\,T_{mb}^3\,\frac{b}{\lambda_a};\quad \varepsilon_{r2} = 4\,\sigma_0\,T_{m2}^3\,\frac{b}{\lambda_a};$$

$$B = 1 - r_2\,F_{22} - 2\,r_1\,r_2\,F_{12}\,F_{21}$$

$$T_{mb} = \frac{T_b + T_\infty}{2};\quad T_{m2} = \frac{T_2 + T_\infty}{2}$$

The efficiency η depends on the fin spacing; if the heat flux from the fin tip is negligible (as it is usually the case) the following expression for η can be used:

$$\eta = \frac{\tanh(m\,L)}{m\,L} \qquad (9)$$

where: $m = \sqrt{\dfrac{2\,h}{\lambda_f\,t}}$

The group $m\,L$ can be written as:

$$m\,L = \sqrt{\frac{3\,Fo\,x}{\sqrt{\dfrac{a}{x^4} + c\,x}}} \qquad (10)$$

where:

$$Fo = \frac{2}{3}\,\frac{L^2\,\sqrt[4]{Ra}\,\lambda_a}{b\,t\,\lambda_f}$$

The non-dimensionalized heat transfer coefficient, $\dfrac{H\,b}{\lambda_a}$, depends on: x, Fo, L', t', R, ε_{rb},ε_{r2}, a_1, a_2.

If the radiative term is small compared to the convective contribution, the factor:

$$K = \frac{\dfrac{H\,b}{\lambda_a}}{2\,L'\,\sqrt[4]{Ra}} \qquad (11)$$

depends only on: x, Fo, L', t'.

In the *figures 3–6* are shown the values of the non-dimensional fins spacing that maximize (6), for various values of the other parameters.

From *figure 6* it can be seen that the radiative component has almost no effect on the values of x_{opt}; *figure 3* shows that for $Fo \leq 2$ the main variable that affects the value of x_{opt} is Fo, while for high values of Fo, the influence of L' becomes important.

In *figure 7*, the values of K, which is proportional to the heat exchanged by the whole finned surface, calculated for $x = x_{opt}$, are displayed as functions of Fo.

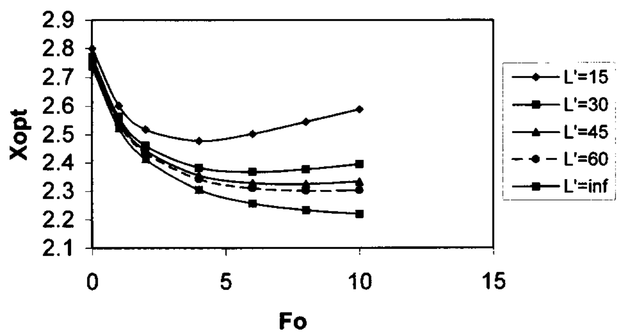

Figure 3. Values of the optimum non-dimensional fins spacing for the various values of Fo and L', with $t' = 0.1$ and $a = 0.8$.

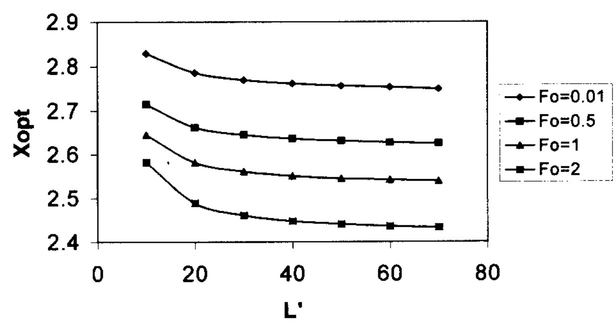

Figure 4. Values of the optimum non-dimensional fins spacing for the various values of L' and Fo with $t' = 0.1$ and $a = 0.8$.

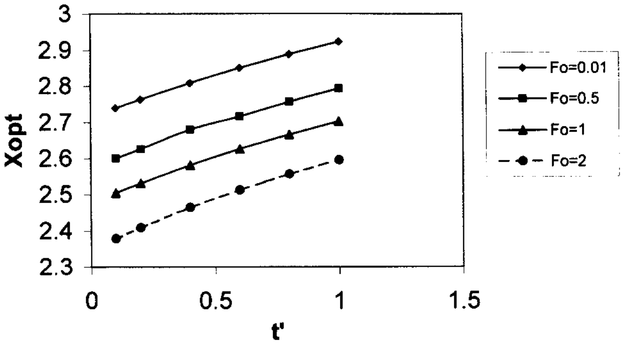

Figure 5. Values of the optimum non-dimensional fins spacing for the various values of t' and Fo with $L' = 30$ and $a = 0.8$.

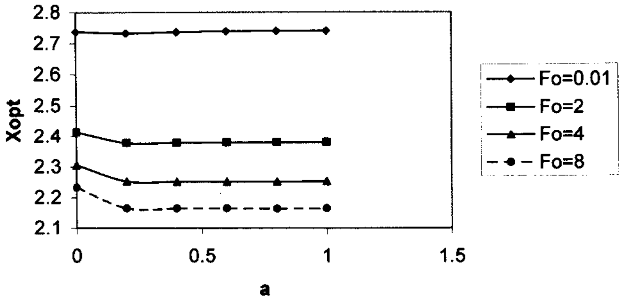

Figure 6. Values of the optimum non-dimensional fins spacing for the various values of a (absorption coefficient) and Fo with $L' = 30$ and $t' = 0.1$.

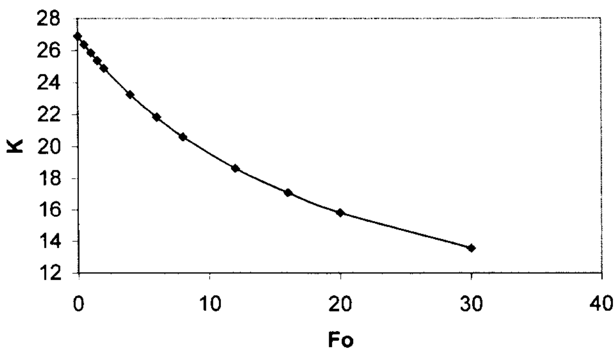

Figure 7. Values of K displayed as functions of Fo, with $L' = 30$ and $t' = 0.1$.

The effect of a finite thermal conductivity of the fin material is a sharp decrease of the x_{opt} value, if compared with the values at $Fo = 0$, that is, at $\frac{\lambda_a}{\lambda_f} = 0$.

This behaviour of x_{opt} as a function of $\frac{\lambda_a}{\lambda_f}$ contrasts with the results obtained in [3], where an increase of the optimum fins spacing for decreasing λ_f is shown in a few plots. However, taking into account the thermal conductivity of the fin material implies that, for any x, the value of the convective term of the overall heat transfer coefficient, calculated for $Fo = 0$, is multiplied by the fin efficiency η, that decreases when Fo increases, and is larger when x is smaller, because, when the fin spacing is narrow, the heat transfer coefficient is small and the fin efficiency is larger.

Thus, comparing two $H - x$ curves, for $Fo \cong 0$ and $Fo > 0$ (*figure 8*), it can be expected that an increase of Fo displaces the position of the maximum to the left.

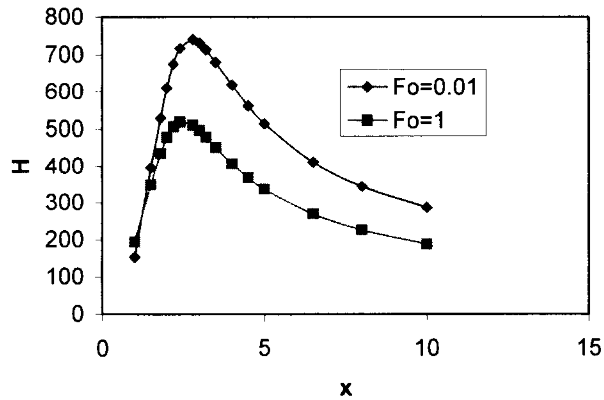

Figure 8. Comparing two $H - x$ curves, for $Fo \cong 0$ and $Fo > 0$, with $L' = 30$, $t' = 0.1$ and $a = 0.8$.

3. SIMPLIFIED RELATIONS

When the product mL is not larger than unity, the fin efficiency can be approximated by the following relation:

$$\eta = \frac{1}{\left[1 + \frac{1}{3}(mL)^2\right]} \quad mL \le 1 \tag{12}$$

In *figure 9* are shown the η values calculated from (12) and (9); it is apparent that (12) is a good approximation for η for $mL \le 1$; it can be observed that, for $x \approx x_{\text{opt}}, mL \approx Fo$, as can be derived from (10).

The dependence of x_{opt} on L' is weak for $Fo \le 1$, so that the second term in (6) can be neglected.

If the radiative contribution and t' are negligible, and the relation (12) is used, a closed form expression of x_{opt} can be obtained:

$$x_{\text{opt}}^6 = \frac{2a}{c\left(1 - \frac{Fo^2}{c^2}\right)} \left[1 + \frac{Fo^2}{4c} - \sqrt{\left(1 + \frac{Fo^2}{4c}\right)^2 - \left(1 - \frac{Fo^2}{c}\right)}\right] \tag{13}$$

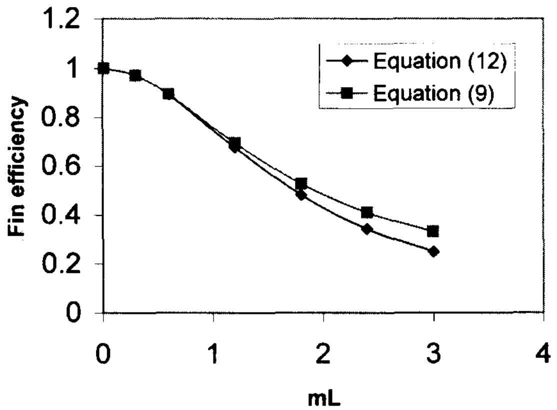

Figure 9. Values of fin efficiency η calculated by (9) and simplified relations (12), shown as functions of the group $m\,L$.

When Fo is of the order of several units, the dependence of $x_{\rm opt}$ on L' cannot be ignored; on the other hand, for $Fo \gg 1$, it can be assumed that:

$$\eta = \frac{\tanh(m\,L)}{m\,L} \cong \frac{1}{m\,L} \tag{14}$$

Using this expression for η, and still neglecting the effect of t' in (6) and the radiative exchange, it is possible to find that $x_{\rm opt}$ is a solution of the equation:

$$x_{\rm opt}^6 = \frac{1}{2}\frac{a}{c} + \frac{3\sqrt{3}}{2}\frac{a}{c}\frac{\sqrt{Fo}}{L'}\frac{x_{\rm opt}}{\sqrt{\dfrac{a}{x_{\rm opt}^6} + c}} \tag{15}$$

If $L' \to \infty$, that is if the heat exchanged by the unfinned portion of the base plate can be neglected, the values of $x_{\rm opt}$ vary from:

$x_{\rm opt} = 2.7155$ for $Fo = 0$ to $x_{\rm opt} = 2.155$ for $Fo \to \infty$.

The first is the well known [1, 3] value of perfectly conducting fins material while the second is the limiting value for fins with zero efficiency.

In *figure 10* the values of $x_{\rm opt}$ calculated from the expressions (9) and (12) of the fin efficiency are compared: it can be seen that (12) is a good approximation for $Fo \leq 1$.

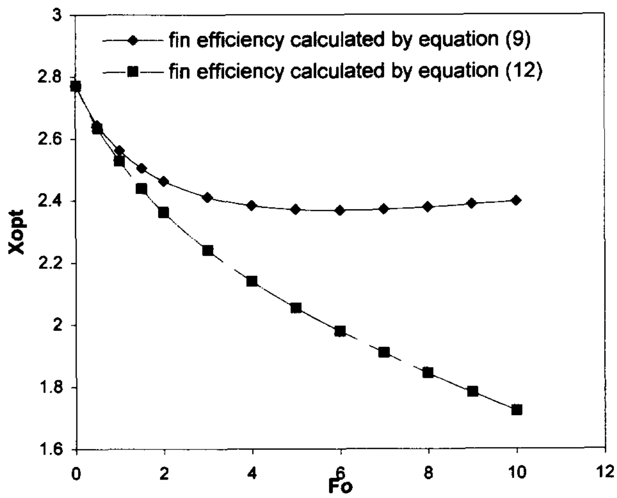

Figure 10. Values of $x_{\rm opt}$ calculated by the expressions (9) and (12), with $L' = 30$, $t' = 0.1$ and $a = 0.8$.

4. CONCLUSIONS

A simple model to calculate the heat transfer of vertical finned surfaces in natural convection has been presented; temperature variations and end effects in the vertical direction have been neglected as well as the temperature dependence of the fluid properties. The effect of the thermal conductivity and of the emissivity of the fins material and of the heat exchanged by the unfinned portion of the base plate on the optimum performance of the system have been evaluated. The main effect of a finite fin conductivity is a reduction of the optimal fins spacing; taking into account this possibility of compacting the system, the heat flux can be increased by as much as 20 %.

Simplified relations for $x_{\rm opt}$ valid in the two limiting cases of very high and very low fin conductivity have been derived; the first is a closed form solution, while the second can be obtained after a few iterations.

REFERENCES

[1] Kraus A.D., Bar-Cohen A., Thermal Analysis and Control of Electronic Equipment, Hemisphere Publ. Corp., Washington 1983.

[2] Jaluria Y., Natural Convective Cooling of Electronic Equipment, in: Kakac et al., Natural Convection. Hemisphere Publ. Corp., 1985, pp. 561–987.

[3] Aihara T. and Maruyama S., Optimum design of natural cooling heat sinks with vertical rectangular fin arrays, in: WIN AUNG Cooling Technology for Eelectronic Equipment, Hemisphere Pub. Corp., Washington 1988, pp. 35–53.

[4] Anand N.K., Kim S.H., Fletcher L.S., The effect of plate spacing on free convection between parallel plates, J. Heat Trans.-T. ASME 114 (1992) 515–527.

[5] Bejan A., Sang W.L., Optimal geometry of convection cooled electronic packages, in: Kakaç S. et al., Cooling of electronic systems, Proceedings of NATO ASI, Kluwer Academic Publishers, 1994, pp. 277–292.